Find the value of $\sum_{n=2}^{\infty} 700(0.5)^{3n-5}$. **HINT: Write out the first few terms first.** **SCORE: _____ / 15 PTS**

$$= 700(0.5) + 700(0.5)^4 + 700(0.5)^7 + \ldots$$

INFINITE GEOMETRIC SERIES

$$r = (0.5)^3 = \frac{1}{8}$$

$$= \frac{700\left(\frac{1}{2}\right)}{1 - \frac{1}{8}}$$

3 4 4

$$= \frac{350}{\frac{7}{8}} = \overset{50}{\cancel{350}} \cdot \frac{8}{\cancel{7}} = 400$$

4

Consider the sequence defined recursively by $a_n = na_{n-1} + \frac{3}{2}n^2 - 3, \quad a_1 = -\frac{9}{2}$. SCORE: _____ / 20 PTS

[a] Find the first 6 terms of the sequence. **Your answers must be integers or fractions, NOT decimal approximations.**

$a_2 = 2a_1 + \frac{3}{2}\cdot 2^2 - 3 = 2(-\frac{9}{2}) + 6 - 3 = \underline{-6}$ 3

$a_1 = -\frac{9}{2}$

$a_3 = 3a_2 + \frac{3}{2}\cdot 3^2 - 3 = 3(-6) + \frac{27}{2} - 3 = \underline{\frac{-15}{2}}$ 3

$a_4 = 4a_3 + \frac{3}{2}\cdot 4^2 - 3 = 4(-\frac{15}{2}) + 24 - 3 = \underline{-9}$ 3

$a_5 = 5a_4 + \frac{3}{2}\cdot 5^2 - 3 = 5(-9) + \frac{75}{2} - 3 = \underline{-\frac{21}{2}}$ 3

$a_6 = 6a_5 + \frac{3}{2}\cdot 6^2 - 3 = 6(-\frac{21}{2}) + 54 - 3 = \underline{-12}$ 3

[b] Based on the first 6 terms, does the sequence appear to be arithmetic, geometric or neither ? Show how you reached your conclusion.

ARITHMETIC 2

$-6 - -\frac{9}{2} = -\frac{3}{2} = d$

$-6 + -\frac{3}{2} = -\frac{15}{2}$

$-\frac{15}{2} + -\frac{3}{2} = -9$

$-9 + -\frac{3}{2} = -\frac{21}{2}$

$-\frac{21}{2} + -\frac{3}{2} = -12$

3

QJ got a new credit card on December 1, 2015, and charged $47 on it that day. On the 1st day of every month after that, QJ charged $13 more than he had charged on the 1st day of the previous month. By December 1, 2017, how much had QJ charged on his card altogether ? (Assume that QJ never charged anything else to his card except on the 1st day of each month.) **SCORE: _____ / 15 PTS**

$$47 + (47+13) + (47+13+13) + \cdots + (47+13\cdot(25-1))$$ ARITHMETIC SERIES

3

$$= \frac{25}{2}(47 + 47 + 13(25-1))$$ 9

$$= 5075$$ DOLLARS

3

Simplify $\binom{2n-1}{2n-4}$.

SCORE: _____ / 15 PTS

$$= \boxed{\frac{(2n-1)!}{(2n-4)!\,3!}} = \boxed{\frac{(2n-1)\overset{n-1}{\cancel{(2n-2)}}(2n-3)\cancel{(2n-4)}!}{\cancel{(2n-4)}!\,3\cdot\cancel{2}\cdot 1}}$$

5 5

$$= \frac{1}{3}(2n-1)(n-1)(2n-3)$$

5

Find a_n for the geometric sequence with $a_3 = 54x^4y$ and $a_6 = -\dfrac{16x^{19}}{y^5}$.

SCORE: _____ / 20 PTS

$$a_n = a_1 r^{n-1}$$

$$a_3 = a_1 r^2 = 54x^4y$$

$$a_6 = a_1 r^5 = -\frac{16x^{19}}{y^5}$$

$$\frac{a_1 r^5}{a_1 r^2} = \frac{\frac{-16x^{19}}{y^5}}{54x^4y} \quad 3$$

$$r^3 = -\frac{8}{27}\frac{x^{15}}{y^6} \quad 4$$

$$r = -\frac{2}{3}\frac{x^5}{y^2} \quad 3$$

$$a_1\left(-\frac{2}{3}\frac{x^5}{y^2}\right)^2 = 54x^4y \quad 3$$

$$a_1 = 54x^4y\left(-\frac{3}{2}\frac{y^2}{x^5}\right)^2$$

$$= \cancel{54}^{27}x^4y \cdot \frac{9}{\cancel{4}_2}\frac{y^4}{x^{10}}$$

$$= \frac{243}{2}\frac{y^3}{\cancel{x^6}} \quad 4$$

$$a_n = \frac{243y^3}{2x^6}\left(-\frac{2x^5}{3y^2}\right)^{n-1} \quad 3$$

Use sigma notation to write the series $-4+25-64+121-\cdots-2500$.

SCORE: _____ / 15 PTS

$$-2^2+5^2-8^2+11^2-\cdots-50^2$$

$$2, 5, 8, 11, \ldots, 50$$

ARITHMETIC SEQUENCE

$$\underbrace{a_n = 2+3(n-1) = 50}_{2}$$

$$3(n-1)=48$$

$$n-1=16$$

$$\underbrace{n=17}_{2}$$

$$3\ \boxed{\sum_{n=1}^{17}} \underbrace{(-1)^n}_{2} \overbrace{\underbrace{(2+3(n-1))}_{3}^2}^{3}$$

$$= \sum_{n=1}^{17} (-1)^n (3n-1)^2$$

Consider the expression $(5x^7 - 11x^3)^{41}$. 2 EACH EXCEPT AS NOTED SCORE: _____ / 25 PTS

[a] Write the first 3 terms of the expansion of the expression. Simplify all exponents.

Your answer may use multiplication and exponents, but NOT division, ! nor ${}_nC_r$ (or equivalent) notation.

$$\underbrace{(5x^7)^{41}}_{1} + \underbrace{\binom{41}{1}(5x^7)^{40}(-11x^3)} + \underbrace{\binom{41}{2}(5x^7)^{39}(-11x^3)^2}$$

$$= 5^{41}x^{287} - 41\cdot 5^{40}\cdot 11x^{283} + \boxed{\frac{41!}{2!39!}}5^{39}11^2x^{279} \qquad \boxed{\frac{41\cdot \overset{20}{\cancel{40}}\cdot 39!}{\cancel{2}\cdot 1\cdot 39!}}$$

$$= \underbrace{5^{41}x^{287}}_{1} \underbrace{-41}_{1}\cdot\underbrace{5^{40}\cdot 11x^{283}}_{1\frac{1}{2}} + \underbrace{41\cdot 20\cdot 5^{39}\cdot 11^2}_{1\frac{1}{2}}\underbrace{x^{279}}_{1\frac{1}{2}}$$

[b] Find the coefficient of x^{211} in the expansion.

Your answer may use multiplication, division, exponents and !, but NOT ${}_nC_r$ (or equivalent) notation.

$$\underbrace{\binom{41}{r}(5x^7)^{41-r}(-11x^3)^r} = \binom{41}{r}5^{41-r}(-11)^r x^{7(41-r)+3r}$$

$$\underbrace{7(41-r)+3r=211}$$
$$287-4r=211$$
$$-4r=-76$$
$$\underbrace{r=19}$$

$$\text{COEFFICIENT} = \underbrace{\binom{41}{19}5^{22}(-11)^{19}}_{1\frac{1}{2}} = \underbrace{-\frac{41!}{19!22!}5^{22}11^{19}}$$

Prove that $\sum_{i=1}^{n}(6i+5)4^{i-1}=(2n+1)4^{n}-1$ for all positive integers n **using mathematical induction**. **SCORE: _____ / 25 PTS**

2 EACH EXCEPT AS NOTED

BASIS CASE: $\underbrace{\sum_{i=1}^{1}(6i+5)4^{i-1}}=\underbrace{11\cdot 4^{0}=11=3\cdot 4^{1}-1}$

INDUCTIVE STEP: ASSUME $\underbrace{\sum_{i=1}^{k}(6i+5)4^{i-1}=(2k+1)4^{k}-1}$ $\underbrace{\text{FOR SOME ARBITRARY INTEGER } k\geq 1}$

$$\underbrace{\sum_{i=1}^{k+1}(6i+5)4^{i-1}}$$

$$=\underbrace{\sum_{i=1}^{k}(6i+5)4^{i-1}+(6k+11)4^{k}}$$

$$=\underbrace{(2k+1)4^{k}-1}+(6k+11)4^{k}$$

$$=\underbrace{(2k+1+6k+11)}4^{k}-1$$

$$=\underbrace{(8k+12)}4^{k}-1$$

$$=\underbrace{4(2k+3)}4^{k}-1=\underbrace{(2(k+1)+1)4^{k+1}-1}$$

SO, $\underbrace{\text{BY MI}}$,

$\sum_{i=1}^{n}(6i+5)4^{i-1}=(2n+1)4^{n}-1$

FOR ALL POSITIVE INTEGERS n